QUANTUM BEAM PHYSICAL INVESTIGATION ON BEAM TRANSVERSE DENSITY DISTRIBUTION

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Abstract

The recently proposed beam physics method was introduced briefly. The intense beam transverse density distribution is investigated and estimated with the quantum beam physical theory by solving the quantum beam dynamic equation of the proton intense beam in free space, It is shown that the estimating results basically agree with the experimental fact, and that the quantum beam dynamics is a new useful method for studying accelerator beam transmission.

1 INTRODUCTION

In the past 50 years, the classical beam dynamic physics achieved great success in designing and constructing accelerator, and itself became a mature and perfect theory in its own right. But the physics development is unending, and no any branch of the physics need not to develop. The accelerator beam physics is also.

For deep going research of the beam transfer characteristics, in recent years, R. Fedele et al. proposed a quantum-like beam physics theory, namely, the *thermal-wave model for relativistic charged particle beam propagation*[1-4] to describe the optics and the dynamics of charged particle beams. R. Jangannathan et al. presented a quantum beam theory based on the Dirac equation[5-7] to study the behavior of electron optics system. Recently *a quantum beam physics method* has been proposed[8, 9]. All of the above beam dynamics theories are for easily solving and explaining a lot of beam phenomena that can not be solved by the classical methods, and the rapidly increasing number of beam phenomena that involve quantum effects, for example, the beams strahlung in the e^+e^- linear collider at energies beyond a TeV, etc.[10, 11]

On the other hand, we find when a proton beam with K-V distribution (energy E=15KeV and current intensity I=52mA) transferred from an aperture (radius R is 0.5cm) to a baffle with a hole in its center, a beam density annular distribution as a diffraction ring forms on the baffle (see fig. 1). It is clear that the beam density is not K-V distribution. Because the de Broglie wavelength λ_d is much less than the beam aperture radius R_0, the particle diffraction is impossible to happen according to the quantum mechanics principle, and the classic beam dynamics also cannot explain this phenomenon. Here we try to explain this phenomenon and to estimate the beam distribution rationally by using the recently proposed beam physics method.

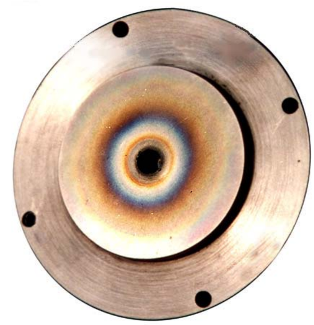


Figure 1: the photograph of beam baffle

2 A BRIEF PRESENTION OF THE QUANTUM BEAM DYNAMICS METHOD

For describing the behaviors of different current intensity beams, we introduced two different forms of quantum beam dynamics master equation

$$\hat{K}\phi(x,y,z)=\{\frac{1}{2\sqrt{2mqV}}(ia\frac{\partial}{\partial x}+qA_x)^2 + \frac{1}{2\sqrt{2mqV}}(ia\frac{\partial}{\partial y}+qA_y)^2-\sqrt{2mqV}-qA_z\}\phi(x,y,z) \quad (1)$$

and

$$(\hat{K}+qA_z)^2\phi(x,y,z)=[2mqV(x,y,z) -(ia\frac{\partial}{\partial x}+qA_x)^2-(ia\frac{\partial}{\partial y}+qA_y)^2]\phi(x,y,z) \quad (2)$$

Here $\phi(x,y,z)$ is the beam propagation wave function, $a=4\pi^2\hbar c=1.25\times10^{-24}\,J\cdot s$, $\hat{K}=ia\partial/\partial z$, q and m are the electric quantity and the mass of beam charged particle, respectively. A_x, A_y and A_z are the magnetic vector potential components of magnetic field, $V(x, y, z)$ is the beam gauge electric potential. For the axisymmetric electromagnetic field, the master equation (1) becomes to the quantum beam dynamics Schrëdinger-like equation

$$\hat{K}\phi(x,y,z)=(\frac{1}{2M^*}\hat{P}^2+U(x,y,z))\phi(x,y,z) \quad (3)$$

where

$$\begin{cases}\hat{P}^2=-a^2(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})\\ U(x,y,z)=\frac{qB(z)}{2\sqrt{2mqV}}l_z+\frac{q^2B(z)^2}{8\sqrt{2mqV}}(x^2+y^2)-\sqrt{2mqV}\\ M^*=\sqrt{2mqV}\end{cases} \quad (4)$$

The Equation (2) can be written as

$$\hat{K}^2\phi(x,y,z)=[2mqV(x,y,z)+a^2(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}) - qB(z)\hat{l}_z-\frac{1}{4}q^2B(z)^2(x^2+y^2)]\phi(x,y,z) \tag{5}$$

The normalized beam propagation function $\phi(x,y,z)$ is in connection with the charged particle beam density distribution at space site (*x*, *y*, *z*). Let us denote by $\sigma(x,y,z)$ and *N* the transverse density number of beam and the total number of particles at beam longitudinal position z, respectively, so the meaning of $\phi(x,y,z)$ is given by the following relationship:

$$\sigma(x,y,z)=N\cdot\left|\phi(x,y,z)\right|^2 \tag{6}$$

Here the normalizing condition of $\phi(x,y,z)$ is

$$\iint_{-\infty}^{+\infty}\phi^*(x,y,z)\cdot\phi(x,y,z)dxdy=1 \tag{7}$$

Furthermore, the beam radius *R*(*z*) is defined as the half width of the beam transverse density, which is the Gaussian distribution:

$$\iint_{-R(z)}^{R(z)}\left|\phi(x,y,z)\right|^2dxdy=0.86 \tag{8}$$

The particle track slope operators $\hat{x}', \hat{y}'$ are defined as

$$\hat{x}'=\hat{P}_x/P_z,\qquad \hat{y}'=\hat{P}_y/P_z \tag{9}$$

In addition, we can demonstrate that Liouville theorem is tenable in quantum beam physics according to the physics annotation of $\phi(x,y,z)$.

3 THE SOLUTION OF QUANTUM BEAM DYNAMICA EQUATION

Suppose that the intense beam is K-V distribution at the position of exit, the charged particle gauge electric potential $V(x,y,z)$ can be written as

$$V(x,y,z)=V_0-\frac{I}{4\pi\varepsilon_0 \mathrm{v}R^2}(x^2+y^2) \tag{11}$$

Where $V_0=m\mathrm{v}_0^2/2q$ is the gauge electric potential at axis *z*, ε_0 is permittivity of free space; v is the velocity of beam particle, *R* is the beam radius.

Substituting equation (10) and equation (11) into equation (5), we find

$$\hat{K}^2\phi(x,y,z)=[a^2(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}) -\frac{mqI}{2\pi\varepsilon_0 \mathrm{v}R^2}(x^2+y^2)+2mqV_0]\phi(x,y,z) \tag{12}$$

For the particle beam moving in the free-drifting space, $\hat{K}$ is a conserved quantity. This means that

$$\hat{K}^2\phi(x,y,z)=P_z^2\phi(x,y,z) \tag{13}$$

and the beam propagation function can be expressed as $\phi(x,y,z)=\varphi(x,y)Z(z)$, the equation (12) becomes

$$\frac{\partial^2}{\partial z^2}Z(z)=-\frac{P_z^2}{a^2}Z(z) \tag{14}$$

and

$$[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\frac{1}{a^2}\frac{mqI}{2\pi\varepsilon_0 vR^2}(x^2+y^2)+\frac{1}{a^2}(2mqV_0-P_z^2)]\varphi(x,y)=0 \tag{15}$$

The solution of equation (14) is

$$Z(z)=A\sin(\frac{P_z}{a}z+\theta)+B \tag{16}$$

Here *A*, *B* and *θ* are decided by the initial beam condition. If we define

$$\frac{1}{a^2}\frac{mqI}{2\pi\varepsilon_0 vR^2}=C^4,\qquad \frac{1}{a^2C^2}(2mqV_0-P_z^2)=\lambda$$

$$X=Cx,\qquad Y=Cy \tag{17}$$

then the equation (15) becomes

$$\left[\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}-(X^2+Y^2)+\lambda\right]\varphi(X,Y)=0 \tag{18}$$

This is a self-conjugate equation, which has the convergence solution under the condition $x\to\pm\infty$ and $y\to\pm\infty$, $\varphi(X,Y)\to 0$. These conditions are enough for researching particle beam. The equation (18) with these boundary conditions is just the Schrëdinger equation of two-dimensional isotropic harmonic oscillator, its solution is

$$\varphi_{m,n}(x,y)=N_{m,n}\exp[-C^2(x^2+y^2)/2]H_m(Cx)H_n(Cy) \tag{19}$$

where $N_{m,n}=C/\sqrt{2^{(m+n)}m!n!\pi}$ is the normalization constant with state number *m*, *n*=0, 1, 2, 3…, and eigenvalue $\lambda=2(m+n+1)$. $H_m(Cx)$ and $H_n(Cy)$ are the Hermitian multinomials. Function $\varphi(x,y)$ has the orthonormality and completeness. The solution of equation (12) can be written as

$$\phi(x,y,z)=\sum_{m,n=0}^{\infty}c_{m,n}\varphi_{m,n}(x,y)Z(z) \tag{20}$$

where $\sum_{m,n=0}^{\infty}c_{m,n}^*c_{m,n}=1$.

4 ESTIMATION TO BEAM TRANSVERSE DENSITY

According to quantum mechanics theory of harmonic oscillator, state *m* and *n* have the following relationship with the transverse position (*x*, *y*), $m=(C^2x-1)/2$ and $n=(C^2y-1)/2$. Submitting definition of *C* into last relationship, we obtain

$$\begin{cases} n=\frac{1}{2a}(\frac{mqI}{2\pi\varepsilon_0 vR^2})^{1/2}x^2-\frac{1}{2} \\ m=\frac{1}{2a}(\frac{mqI}{2\pi\varepsilon_0 vR^2})^{1/2}y^2-\frac{1}{2} \end{cases} \tag{21}$$

For the proton beam with *E*=15KeV, *I*=52mA and *R*=0.5cm, its first state number *m* and *n* corresponding transverse position (*x*, *y*) is shown in table 1.

According to the definition of beam transverse density, we find

$$\sigma(x,y,z) = N \sum_{n=m=0}^{\infty} c_n \left|\varphi_{n,n}(x,y,z)\right|^2 \qquad (22)$$

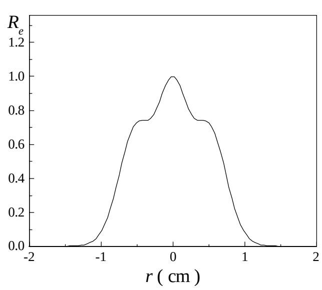
where $\sum_{n=0}^{\infty} c_n = 1$.

Table 1.

State number	Transverse position
m=n=0	x=y≈4.037mm
m=n=1	x=y≈6.993mm
m=n=2	x=y≈9.028mm
m=n=3	x=y≈10.68mm

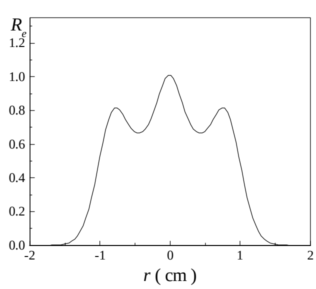
We find that the initial beam radius R is located between the positions decided by the state m=n=0 and state m=n=1. With supposing that the beam particle density decreases rapidly with the beam radius increasing and submitting equation (19) into equation (22), we can obtain the beam particle transverse density distribution, which relative density R_e ($R_e = \sigma(x,y,z)/\sigma(0,0,z)$) is shown in Fig. 2.

Comparing the fig. 1 and fig. 2, we find that the higher the state number m and n is, the more closely the estimating distribution of beam transverse density would agree with the fact.



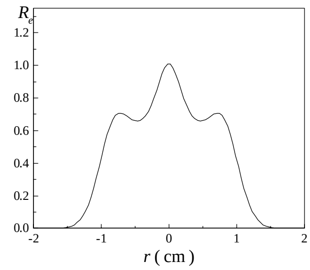
m=n=0, 1. c_0=0.8, c_1=0.2

(a)



m=n=0, 1, 2. c_0=0.8, c_1=0.16 , c_2=0.04

(b)



m=n=0, 1, 2, 3. c_0=0.8, c_1=0.16 , c_2=0.03 , c_3=0.01

(c)

Figure 2: Curves of the beam relative density R_e distribution with the beam transverse position r in different state number m and n. Here $R_e = \sigma(x,y,z)/\sigma(0,0,z)$.

5 DISCUSSION

It is difficult to compute the beam particle distribution in classic beam dynamics. Here we use the recently proposed beam physics method to estimate the beam transverse particle distribution, and find that the estimating results basically agree with the experimental fact. Therefore, we can consider that the quantum beam dynamics maybe is a new useful method in studying accelerator beam transmission.

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