COHERENT SYNCHROTRON OSCILLATION INDUCED BY A PLL NOISE IN THE KEK PHOTON FACTORY STORAGE RING

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Abstract

In the 2.5-GeV Photon Factory (PF) storage ring at KEK, a coherent synchrotron oscillation of the stored bunches has been observed. This oscillation appears in a beam spectrum as the synchrotron sidebands beside rf harmonics, or it can be observed directly using a streak camera. It emerges even under a very low beam current of 1 mA or less, and its amplitude becomes small at high currents. Therefore, this oscillation could not be attributed to any beam instabilities.

We have recently identified that this oscillation was mainly caused by a phase noise in an rf voltage. The noise was produced by a phase-lock loop (PLL) circuit in a low-level rf system. We report both an experimental investigation and a cure for this oscillation.

1 SYNCHROTRON OSCILLATION INDUCED BY AN RF PHASE NOISE

When an rf phase is modulated by some noise, it can drive synchrotron oscillations of the stored electrons. This effect has been discussed, for example, in reference [1]. We review some of the results here.

An equation of a small synchrotron oscillation under phase modulation $\phi_m(t)$ due to random noise is given by

$$\ddot{\phi} + 2\lambda\dot{\phi} + [\omega_s^2 + g(t)]\phi = f(t)\,, \qquad (1)$$

with

$$f(t) = \omega_s^2 \phi_m(t)\,,\ \ g(t) = -\omega_s^2 \cot\phi_0 \cdot \phi_m(t)\,, \qquad (2)$$

where ϕ is the particle phase relative to the synchronous particle, λ the radiation damping rate, ω_s the angular synchrotron frequency, ϕ_0 the synchronous phase (defined by $\cos\phi_0 = U_0/(eV_c)$, where U_0 is the radiation loss per turn and V_c the total rf voltage), and the dot denotes the derivative by time. We take the positive sign for the phase delays. When the phase modulation contains broadband spectrum, the $g(t)$ term is less important than the $f(t)$ term, and thus, we can neglect it.

In order to ensure the convergence of the Fourier integrals used below, we take a long period $2T$, and replace $f(t)$ by its truncated function $f_T(t)$, which is defined by

$$f_T(t) = f(t) \ \ (\text{for } |t| \le T\,);\ \ f_T(t) = 0 \ \ (\text{for } |t| > T\,). \qquad (3)$$

Then, we investigate the following equation of motion:

$$\ddot{\phi} + 2\lambda\dot{\phi} + \omega_s^2\phi = f_T(t)\,. \qquad (4)$$

By applying the Fourier transform, a stationary solution of Eq. (4) is given by

$$\phi(t) = \int_{-\infty}^{\infty} H(\omega)F(\omega)e^{i\omega t}d\omega\,, \qquad (5)$$

where $F(\omega)$ is the Fourier transform of $f_T(t)$, and $H(\omega)$ the transfer function, both of which are defined by

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f_T(t)e^{-i\omega t}dt\,,\ \ H(\omega) = \frac{1}{\omega_s^2 - \omega^2 + 2\lambda\omega i}\,. \qquad (6)$$

Due to a resonant form of $H(\omega)$, the frequency components of $F(\omega)$ at around the synchrotron frequency mainly contribute to the synchrotron oscillation. The phase-modulation at a frequency of $\omega = \omega_s$ results in a Q-times larger phase oscillation, where $Q = \omega_s/(2\lambda)$ is the Q-value of the oscillator.

We imagine N sets of similar systems, and consider an average of ϕ^2 over such ensembles. From Eq. (5), we have

$$\left\langle \phi^2(t) \right\rangle = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' H(\omega)H(\omega')\left\langle F(\omega)F(\omega') \right\rangle e^{i(\omega+\omega')t}\,, \qquad (7)$$

with

$$\left\langle F(\omega)F(\omega') \right\rangle = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \left\langle f_T(t) f_T(t') \right\rangle e^{-i(\omega t+\omega' t')}\,. \qquad (8)$$

Assuming that the system is stationary, the above $\left\langle f_T(t) f_T(t') \right\rangle$ should depend only on the time difference $\tau = t' - t$. Then, we define the autocorrelation function of $f_T(t)$ by

$$R_f(\tau) \equiv \left\langle f_T(t) f_T(t+\tau) \right\rangle\,. \qquad (9)$$

Integrating Eq. (8) yields

$$\left\langle F(\omega)F(\omega') \right\rangle = S_f(\omega)\delta(\omega+\omega')\,, \qquad (10)$$

where $S_f(\omega)$ is the spectral density which is defined by

$$S_f(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_f(\tau)e^{-i\omega\tau}d\tau\,. \qquad (11)$$

Substituting Eq. (10) into Eq. (7) yields [1]

$$\left\langle \phi^2 \right\rangle = \int_{-\infty}^{\infty} |H(\omega)|^2 S_f(\omega)d\omega\,, \qquad (12)$$

which is independent of t. When the system is both stationary and ergodic (i.e., the $f_T(t)$ takes its all possible values within a very long period), we can replace the ensemble average by a time average over the period of $2T$ [2]:

$$\left\langle f_T(t) f_T(t+\tau) \right\rangle = \left\langle f_T(t) f_T(t+\tau) \right\rangle_T\,. \qquad (13)$$

In such a case, the spectral density is expressed by the Fourier transform of $f_T(t)$:

$$S_f(\omega) = \frac{\pi}{T}|F(\omega)|^2\,. \qquad (14)$$

2 EXPERIMENT

In the PF storage ring, four rf cavities provide a total rf voltage of 1.7 MV. Each cavity is driven by an independent klystron. A phase of each klystron output is locked to master rf, using a phase-lock loop (PLL). We recently found that a klystron output signal contains some phase noise, which is produced by the PLL circuit. Figure 1 shows the spectra of the picked-up rf signals from one of the cavities; the upper trace shows the spectrum under usual condition (PLL on), while the lower trace shows the one under PLL off. We can see a broadband noise around the rf signal when the PLL was on.

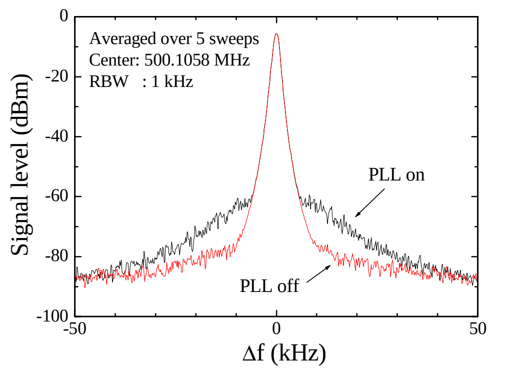

Figure 1: Spectra of the picked-up signals from the cavity.

Figure 2 shows a phase detector circuit[3] used for the PLL. Two rf signals are converted down to 500 kHz, while keeping the phase difference between them. These signals are converted to square-wave signals using a zero-crossing detector, and then, the phase difference is detected using both a digital flip-flop and a low-pass filter. After some amplification, the output signal is fed back to a phase shifter. Linearity of the output voltage to the phase difference is excellent, however, the output signal may contain some white noise due to the square wave if the low-pass filtering at the end is not sufficient. In our case, a broadband phase-noise due to this would induce the coherent synchrotron oscillation, which had been observed in the PF storage ring.

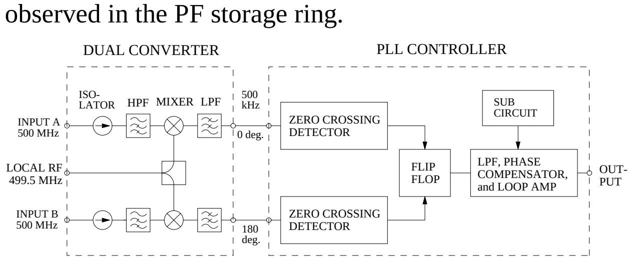

Figure 2: Phase detector circuit used for the PLL.

An experiment was carried out in the PF storage ring under single bunch operation with low beam currents (2.3 - 3 mA). The principal parameters of the ring are given in Table 1. Figure 3 shows the beam spectra from a button-type monitor around the rf frequency; the upper trace was taken under an usual condition. We can see the synchrotron sidebands showing the coherent synchrotron oscillation; these sidebands were lower by about 41 dB from the rf peak. The lower trace in Fig. 3 shows the beam spectrum when we inserted a passive low-pass filter (LPF) between the phase detector and the phase shifter in each rf station. The LPF was an LC-filter having a cutoff frequency of 4.8 kHz. We also reduced a loop gain from 39 dB to 31 dB. As a result, the synchrotron sidebands reduced by about 19 dB. In other words, by eliminating the high-frequency components of the noise, the coherent oscillation was reduced by an order of magnitude. This result clearly showed that the coherent synchrotron oscillation was induced mainly by the phase noise which was produced by the PLL circuits.

Table 1: Principal parameters of the PF storage ring.

Beam energy (E_0)	2.5 GeV
RF frequency (f_{rf})	500.10718 MHz
Total rf voltage (V_c)	1.7 MV
Momentum compaction (α)	0.0061
Synchrotron frequency (f_s)	23.6 kHz
Longitudinal damping rate (λ)	255 s^{-1}
Natural rms bunch length (σ_τ)	31 ps

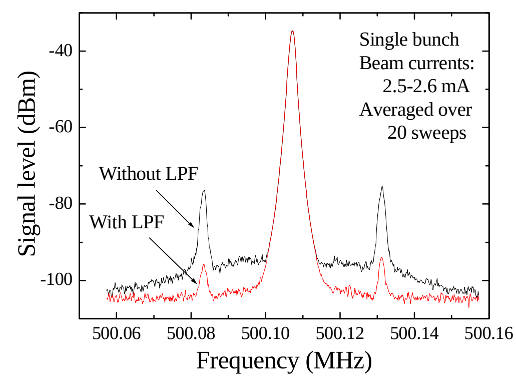

Figure 3: Spectra of the button-type electrode signal.

An evolution of the coherent synchrotron oscillation $\phi(t)$ under usual conditions, which was measured by detecting the phase of the button-type monitor signal, is shown in Fig. 4. The upper trace shows the time evolution; the lower trace is the Fourier transform of it. Because the oscillation was driven by the random noise, the oscillation amplitude fluctuated. Its Fourier transform [Fig. 4(b)] showed a peak at the synchrotron frequency (f_s) of 23.6 kHz. On the other hand, a similar measurement under noise reduction (Fig. 5) showed much smaller synchrotron oscillation. The peak at the synchrotron frequency [in Fig. 5(b)] reduced more than an order of magnitude; however, phase oscillations at low frequency increased to some extent due to an reduced feedback gain.

Figure 6 shows the spectra of the phase noise in one of the cavities, which was detected using a double-balanced mixer. High-frequency phase noise, which can be seen in Fig. 6(a), decreased considerably after the noise reduction [Fig. 6(b)]. Rms amplitudes of the phase noise were about 5.8 mrad and 3.3 mrad before and after the noise

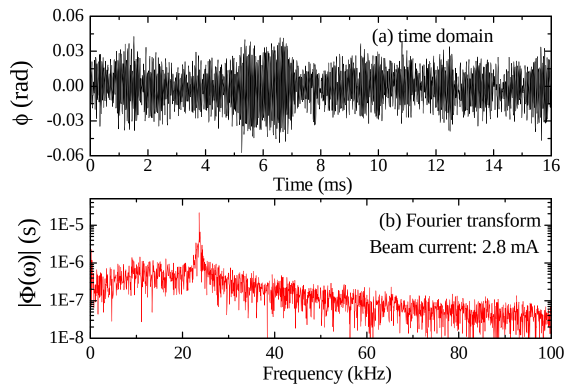

Figure 4: Measured synchrotron oscillation under usual PLL conditions (without LPF's, loop gain: 39 dB).

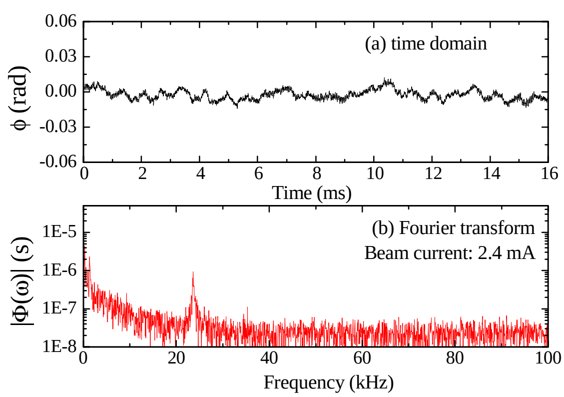

Figure 5: Measured synchrotron oscillation after noise reduction (with LPF's, loop gain: 31 dB).

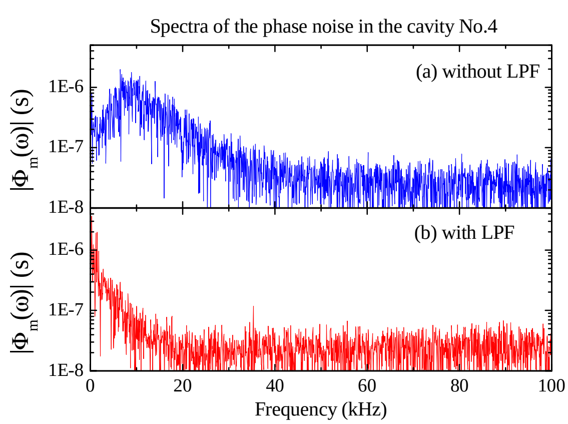

Figure 6: Measured spectra of the phase noise in one of the cavities. (a) Without LPF, (b) with LPF.

reduction, respectively.

It is useful to compare the above experimental results with the theoretical analysis given in Sec. 1. The measured synchrotron-oscillation amplitude σ_ϕ (defined by $\sigma_\phi^2 \equiv \langle \phi^2(t) \rangle_T - \langle \phi(t) \rangle_T^2$) was 16 mrad from Fig. 4(a), and was 19 mrad from an average over ten such traces, respectively. On the other hand, an estimated rms amplitude, using Eqs. (12) and (14) with the measured phase noise, was 31 mrad from Fig. 6(a), and was 36 mrad from ten such traces, respectively. We should note here that the synchrotron oscillation and the cavity phase noise were recorded at the same time and that the cavity phase was detected only from one of the four cavities. The estimated amplitude was about two-times larger than the measured one. If the phase noises of the four cavities were not correlated to each other, an estimated rms synchrotron amplitude should be a half of the above estimation (i.e., 16 mrad and 18 mrad, respectively). Taking this into account, the estimated rms amplitude agrees well with the measured synchrotron amplitude.

Figure 7(b) shows the estimated Fourier spectrum of the synchrotron oscillation, using Eq. (5) with the phase-noise spectrum of Fig. 6(a). This spectrum is very similar to the measured one [Fig. 4(b)] except in high frequency range, where the measurement would be limited by a small noise.

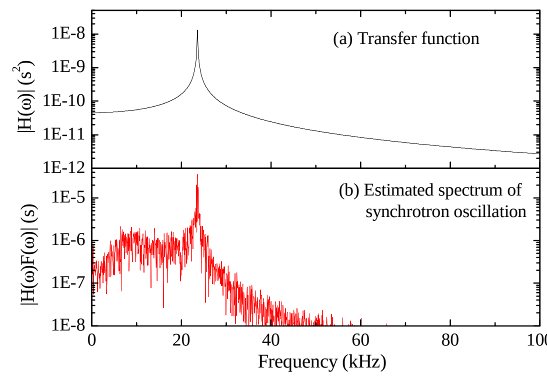

Figure 7: Synchrotron oscillation spectrum calculated from the phase noise of Fig. 6(a).

3 CONCLUSIONS

We have shown experimentally that the coherent synchrotron oscillation of a bunch in the PF storage ring was mainly induced by an rf phase noise, which was produced by the PLL circuits. By both inserting low-pass filters at the end of the PLL circuits and reducing the feedback gain slightly, the coherent oscillation could be reduced. The observed amplitude of the synchrotron oscillation agreed with the one estimated from the measured phase noise.

Under usual high-current operations, there has been few problems due to this oscillation because it is damped strongly by the Robinson damping effect. Under single-bunch operation, on the other hand, the beam performance will be improved by reducing this oscillation since the Robinson damping is moderate.

REFERENCES

[1] K.W. Ormond and J.T. Rogers, PAC'97, p. 1822.

[2] F. Reif, *Fundamentals of statistical and thermal physics*, Chapter 15, McGraw-Hill, 1965.

[3] H. Kobayakawa and Y. Yamazaki, "The Tuner Control System for the PF Cavity", KEK Report, No. KEK 83-9, 1983.