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STUDY OF COHERENT TUNE SHIFT CAUSED BY ELECTRON CLOUD IN POSITRON STORAGE RINGS

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Abstract

We discuss the transverse betatron tune shift of the coherent dipole motion of a beam interacting with an electron cloud. A positron beam which passes through a frozen charge distribution experiences an electric field determined by the Coulomb law, and its tune shifts in the positive direction. The electrons in the actual cloud are not frozen, but move during the bunch passage. Thus, the electron distribution varies due to the interaction with the beam and it is "soft". We study the dipole tune shift of the beam interacting with such a "soft" charge distribution.

1 INTRODUCTION

In positron storage rings, an electron cloud can be formed by photoemission and secondary emission, if the ring is operated with a long bunch train (more than 10 bunches) and short bunch spacing (< 10ns). The electron density first increases along the bunch train and then saturates due to the electron space charge field. The electron cloud induces a tune shift of the positron beam. At KEKB the measured tune shift along the bunch train increases both in the horizontal and vertical plane [1]. In this report, we discuss the tune shift caused by the electron cloud using analytic and numerical methods. The tune shift is naively estimated by calculating the electric field for a frozen electron distribution. The electric field at $\boldsymbol{r} = (x, y)$ for a uniform charge distribution with cylindrical symmetry is

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{\rho_e}{2\epsilon_0}\boldsymbol{r}, \tag{1}$$

where ρ_e is the electron cloud density. The electric focusing force induces a tune shift

$$\Delta\nu_{x(y)} = \frac{r_e}{2\gamma}\langle\beta_{x(y)}\rangle\rho_e\ L, \tag{2}$$

where γ and $\langle\beta\rangle$ are the relativistic factor and the average beta function of the ring, respectively. For a 'flat' charge distribution with horizontally planar symmetry, there is neither a horizontal field nor a horizontal tune shift, but the vertical field and vertical tune shift are two times stronger than those given in Eq.(1) and Eq.(2), respectively.

A first estimate of the tune shift can be obtained by assuming that the electron cloud distribution is fixed, with the result above. However, the electron cloud is strongly disturbed by the beam force, suggesting that the tune shift could be significantly different from that expressed by Eq.(2). For example, Fig. 1 illustrates the interaction of beam and an electron of the cloud. The electron oscillates in the electric field of bunch. The net force may cancel after integration in the longitudinal direction, and in this case the electron does not affect to the coherent centroid motion of the beam. This means that a more careful consideration may be required in order to estimate the coherent tune shift.

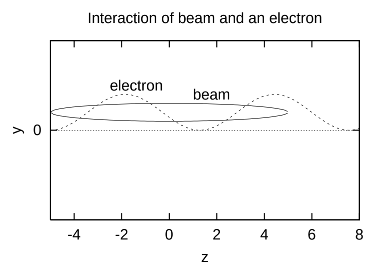

Figure 1: Motion of an electron interacting with beam.

2 ANALYTIC APPROACH

We next consider an electron cloud with a Gaussian distribution of the same fixed transverse rms size as the beam. In this refined model the electron cloud can have a dipole moment in the transverse plane. This means that one degree of freedom for the transverse dipole motion is added to the completely frozen distribution of Eq.(1). As before, the beam is assumed to have a uniform flat top distribution in the longitudinal direction: that is, the charge line density is considered to be constant between $-\sigma_z < z < \sigma_z$, where σ_z is the half bunch length. The electron cloud is spread out uniformly between $0 < s < L$, L denoting the ring circumference.

We discuss only the vertical motion. The extension to the horizontal motion is straightforward. The equation of motion for the beam is [2]

$$\frac{d^2 y_b(s,z)}{ds^2} + \left(\frac{\omega_\beta}{c}\right)^2 y_b(s,z) = \frac{\lambda_b r_e}{\gamma}\int_z^{\sigma_z} W_1(z-z')y_b(s,z')dz'. \tag{3}$$

W_1, which is called the wake field, is

$$W_1(z)[\mathrm{m}^{-2}] = \frac{\lambda_e}{\lambda_b}\frac{L}{(\sigma_x+\sigma_y)\sigma_y}\omega_e c \sin\left(\frac{\omega_e}{c}z\right), \tag{4}$$

where λ_e and λ_b are the line densities of cloud and beam, and σ_x and σ_y are horizontal and vertical beam sizes, re-

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spectively. ω_e is the angular oscillation frequency of electrons interacting with the beam,

$$\omega_e^2 = \frac{\lambda_b r_e c^2}{(\sigma_x + \sigma_y)\sigma_y}. \tag{5}$$

Second term of Eq.(4) is a betatron angular frequency corrected by electric field of electron cloud,

$$\omega_\beta = \omega_{\beta 0} + \frac{\lambda_e r_e c^2}{2\omega_{\beta 0}\gamma(\sigma_x + \sigma_y)\sigma_y}. \tag{6}$$

If we consider only Eq.(6), the tune shift is the same as Eq.(2). Including also the effect of the wake field, the tune shift of the coherent dipole mode becomes [3]

$$\Delta\omega_{\beta,\mathrm{coh}} = \frac{\omega_b^2}{2\omega_\beta} - i\frac{\lambda_e \sigma_z r_e c}{\gamma T_0 \omega_\beta}\int_{-\infty}^{\infty} d\omega Z_1(\omega) g_{00}(\omega)^2 \tag{7}$$

where g_{00} is a function referring to an unperturbed distribution in the longitudinal phase space, and $Z_1(\omega)$ is impedance due to the electron cloud, which is the Fourier transform of the wake field. The imaginary part of Z_1 which contributes to the tune shift is expressed by,

$$\mathrm{Im}Z_1(\omega) = \frac{\lambda_e L}{\lambda_b c(\sigma_x + \sigma_y)\sigma_y}\left(\frac{1}{\omega - \omega_e} + \frac{1}{\omega + \omega_e}\right). \tag{8}$$

For a uniform beam of $-\sigma_z < z < \sigma_z$, g_{00} is

$$g_{00} = \frac{1}{2}\sqrt{\frac{c}{\omega\sigma_z}} J_{1/2}\left(\frac{\omega\sigma_z}{c}\right) = \frac{1}{\sqrt{2\pi}}\frac{c}{\omega\sigma_z}\sin\frac{\omega\sigma_z}{c}. \tag{9}$$

Performing the integral of Eq.(7), the tune shift is

$$\Delta\omega_{\beta,\mathrm{coh}} = \frac{\omega_b^2}{2\omega_\beta}\frac{c}{2\omega_e\sigma_z}\sin\left(\frac{2\omega_e\sigma_z}{c}\right). \tag{10}$$

In the short bunch limit ($\sigma_z \to 0$), the tune shift is

$$\Delta\omega_{\beta,\mathrm{coh}} = \frac{\omega_b^2}{2\omega_\beta}. \tag{11}$$

This is the same as Eq.(2). Since the electrons do not move during the passage of the short bunch, in this case the tune shift is determined by the static charge distribution.

For a longer bunch, the tune shift oscillates and decreases. In the extremely long bunch limit we obtain

$$\Delta\omega_{\beta,\mathrm{coh}} = 0. \tag{12}$$

This behavior is consistent with our image drawn in Fig. 1.

3 NUMERICAL APPROACH

We estimate the tune shift using a numerical method. The simulation for the calculation of the wake force is performed following the same procedure as was used for studying a short range wake field of electron cloud in Ref. [2]. We consider an electron cloud with a transverse size represented by macro-particles and a micro-bunch train along z with a very narrow spacing. We can simulate an electron cloud of arbitrary distribution and with any size of the macro-particles.

To estimate the wake field, in Ref. [2] we have calculated the kicks which the micro-bunches experience from the electron cloud, when the "first" micro-bunch passes through the cloud with a finite transverse displacement smaller than the rms beam size. On the other hand, to estimate the tune shift we now calculate the kicks, when "all" micro-bunches are displaced by an equal amount with respect to the center of the electron cloud. The tune shift is obtained by integrating the kicks along z. We estimate the tune shift for the parameters of KEKB : i.e., $\sigma_x = 420\mu$m, $\sigma_y = 60\mu$m, $N_+ = 3.3 \times 10^{10}$, and $\sigma_z = 5$mm. The cloud density is assumed to be $\rho_e = 10^{12}$ m^{-3}. The tune shift linearly depends on the cloud density, since all electrons are assumed to move independently (we ignore the space charge force acting between them, which is small compared with the attractive beam force). The calculations are performed for various sizes of the electron cloud with an initially Gaussian distribution. The cloud size is characterized by (Σ_x, Σ_y), quoted in unit of the beam size (σ_x, σ_y). For example, in this convention the electron line density λ_e for (a, b) is $a \times b$ times that of (1,1).

We begin the discussion with the vertical tune shift. Figure 2 shows the vertical kicks which micro-bunches experience for various cloud sizes. For (1,1), the kicks oscillate like a cosine function. Their amplitude and frequency are consistent with the analytical expression. Integrating the kicks along the bunch, we get the same tune shift as in Eq.(10). Hence, for a small cloud size the analytical formula of Eq.(10) is reproduced by the numerical method.

Increasing the cloud size, the feature of the kicks changes substantially. For (5,5), the kicks still have a sinusoidal shape, but their oscillation frequency decreases. For (20,20) and (50,50), the kicks are nearly constant. The kick which the first micro-bunch experiences is the same as that in Eq.(1), that is, the tune shift is expressed by Eq.(2), or its equivalent for non-circular symmetry. Contrary to our conjecture, the result agrees with that for a frozen distribution. The value of the kick saturates for a cloud size around (20,20). In contrast, the vertical wake field saturates for increasing cloud size already at a value of about (5,5) [2]. This demonstrates that electrons at larger amplitudes contribute to the tune shift. Electrons with large amplitudes have a much slower oscillation frequency than those near the beam center. In particular, they almost do not move during a bunch passage and give rise to a static force acting on the beam. We have also calculated the kick for (20,50). It is 10% smaller than those of (20,20) and (50,50). This result is peculiar; that is, the kick of (20,50) is less, though the cloud size is larger than for (20,20). A similar variation with the cloud aspect ratio is seen in the horizontal plane.

We next discuss the horizontal tune shift. Figure 3 shows the horizontal kick which micro-bunches experience for various cloud sizes. For (1,1), the kick is consistent with the analytical evaluation. Increasing the size of the electron cloud, the kick seems to saturate for (10,10), (20,20)

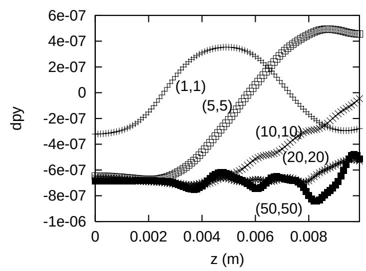

Figure 2: Vertical kick experienced by micro-bunches with a finite displacement ($\Delta y = 50\mu$m) vs. longitudinal position z. The vertical bunch size is $\sigma_y = 60\mu m$. The five lines correspond to various cloud sizes as indicated.

and (50,50) at first sight. However decreasing the horizontal size to (20,50) or (10,50), the kick increases further. This behavior can be understood by considering the macroscopic shape of the electron cloud. The beam has an aspect ratio of $\sigma_x/\sigma_y = 7$. The cloud for (10,10), (20,20) and (50,50) has the same aspect ratio, $\Sigma_x/\Sigma_y = 7$. The horizontal electric field is weak for such a large aspect ratio. For (10,50), the aspect ratio gets closer to 1, with the result that the horizontal electric field become stronger. The behavior of the vertical kick discussed above can also be understood by the same reasoning.

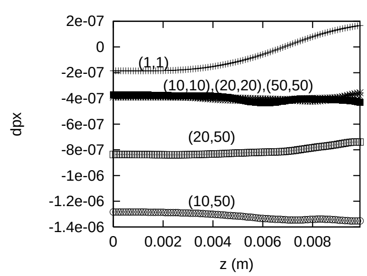

Figure 3: Horizontal kick experienced by micro-bunches with a finite displacement ($\Delta x = 200\mu$m) vs. longitudinal position z. The horizontal bunch size is $\sigma_x = 420\mu m$. The six lines correspond to various cloud sizes as indicated.

Figure 4 shows the horizontal kicks computed when only the first micro-bunch is displaced. The figure shows that the kick which the first micro-bunch experiences strongly depends on the size (shape) of the cloud, while the wake force acting on the following micro-bunches is almost independent of the size. The kick which the first micro-bunch experiences is nothing but the tune shift. Further calculations confirm that the first kick (tune shift) strongly depends on the cloud aspect ratio, but not on the size. On the other hand, the wake field (kicks on succeeding microbunches) depends neither on the size nor on the aspect ratio of the cloud, if the size is larger than a certain value ($r > 5\sigma_x$).

From these results, we can conjecture the main features of the beam-electron cloud interaction. Electrons far from the beam do not move during the passage of a bunch. The electric field which a bunch experiences is determined by the macroscopic structure and/or the symmetry of the electron cloud, and it is dominated by electrons at large amplitudes. Therefore, the kicks in Figs. 2 and 3 are almost constant for all the micro-bunches. Since the electric field causes the tune shift of the beam, the latter is determined by the electron distribution at large amplitudes. The tune shift can be estimated from the simple Eq.(2), or its equivalent for non-circular symmetry. On the other hand, the electrons far from the beam do not contribute to the head-tail wake force, since they do not move during a bunch passage. The wake force is determined by the electrons located near the beam, in a cylindrical region of a few times σ_x [2].

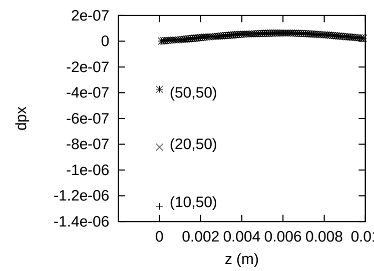

Figure 4: Horizontal kick vs. longitudinal position, when only the first micro-bunch is displaced, by $\Delta x = 200\mu$m. The four lines refer to various cloud sizes as indicated.

4 SUMMARY

We have discussed the tune shift of a beam interacting with an electron cloud. Naively, this tune shift is computed for a frozen electron distribution. However, the electron cloud is soft and changes its shape during the passage of the bunch. As a result, if the electron cloud is of the same size as the beam, the tune shift is reduced and approaches zero for long bunches. We have studied numerically the contribution of electrons far from the beam, which do not move during the bunch passage. Our important result is that the tune shift is determined by these electrons. Thus, it is due to the macroscopic structure of the electron cloud, for cloud sizes much larger than the beam. In this case, we can estimate the tune shift using the naive formula. Our final result is simple but its derivation is not straightforward.

5 ACKNOWLEDGMENTS

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6 REFERENCES

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