

## SHORT RANGE WAKE FIELD CAUSED BY ELECTRON CLOUD IN BENDING MAGNET

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### Abstract

A short-range wake field caused by the electron cloud has previously been studied for a drift space. In a bending magnet, the cloud electrons undergo cyclotron motion with a small radius (<1mm) and at a high frequency (>10GHz) in the horizontal plane due to the strong magnetic field of order 1T. In this report, we study the motion of electrons under the combined influence of a strong magnetic dipole field and the electric field of the beam on the time scale of the bunch length, discuss the short-range wake field caused by the electrons, and simulate the emittance growth. As expected, the wake field in a bending magnet is very different from that in a drift space. The dipole field almost completely suppresses any horizontal coherent motion and rms-size blow-up, and it also slows down the instability in the vertical direction.

### 1 INTRODUCTION

In positron and proton storage rings, an emittance growth has been observed in multi-bunch operation with close spacing ( $\approx 10$  ns) [1, 2, 3]. As a possible explanation, a single bunch instability caused by an electron cloud has been discussed [4, 5]. The wake force of the electron cloud in a field free case has been studied in reference [6]. This paper discusses the effect of the photoelectron cloud in a bending magnet, including an evaluation of the transverse wake and the mode coupling caused by the electron cloud.

### 2 WAKE FIELD CAUSED BY ELECTRON CLOUD IN DIPOLE MAGNET

The photoelectron cloud induces a short-range wake field. When the head part of the bunch is disturbed, this wake will affect the tail of the bunch. Such a short-range wake field excites beam break up (BBU) in a linac and mode coupling in a storage ring. In this paper, the wake force induced by the electron cloud is calculated for a region with dipole magnet. A coasting beam is assumed for evaluating the wake field, which means that the charge distribution of the beam is taken to be uniform in the longitudinal direction. The wake field is evaluated from a numerical program. The electron cloud density at the pipe center  $\rho_c$  is chosen to be  $10^{12} \text{ m}^{-3}$ , a typical value.

We estimate the wake force induced by the electron cloud using a computer simulation. The simulation for the calculation of the wake force is performed following the same procedure as was used for studying the multi-bunch

electron-cloud instability in Ref. [7]. We consider an electron cloud with a finite transverse size, represented by macro-particles, and a micro-bunch train with a very narrow spacing. Note again that the micro-bunch train represents a 'coasting beam', i.e., a bunch with uniform longitudinal charge distribution. The motion of the macro-particles in the electron cloud is expressed by

$$\frac{d^2 \mathbf{x}_{e,a}}{dt^2} = -\frac{2N_+ r_e c}{N_b} \sum_{i=1}^{N_p} \mathbf{F}_G(\mathbf{x}_{e,a} - \bar{\mathbf{x}}_{p,i}; \sigma) \delta(t - t(s_b)), \quad (1)$$

where the force  $\mathbf{F}_G(\mathbf{x})$  is expressed by the Bassetti-Erskine formula [8] normalized so that  $\mathbf{F}_G(\mathbf{x}) \rightarrow \mathbf{x}/|\mathbf{x}|^2$  as  $|\mathbf{x}|/|\sigma| \rightarrow \infty$ .  $N_b$  and  $\sigma$  are the number of particle in a micro-bunch and transverse beam size, respectively.

When micro-bunches pass through the center of the cloud, they are not affected by the cloud and also the center of mass of the cloud does not move. If a micro-bunch with a small transverse displacement passes through the cloud, the cloud is perturbed and its center of mass changes. The subsequent micro-bunches are deflected by the perturbed cloud:

$$\Delta \bar{\mathbf{x}}'_{p,j} = -\frac{2r_e}{\gamma} \sum_{i=1}^{N_e} \mathbf{F}_G(\bar{\mathbf{x}}(s)_{p,i} - \mathbf{x}_{e,a}; \sigma) \quad (2)$$

From Eq.(2), the wake force is calculated as the response to a small displacement of a micro-bunch,  $\bar{\mathbf{x}}_{p,i} = \Delta \mathbf{x}$ ,

$$W_1(z_i - z_j) = \frac{\gamma}{N_b r_e} \frac{\Delta \bar{\mathbf{x}}'_{p,i}}{\bar{\mathbf{x}}_{p,i}} \quad \text{for } z_i > z_j \quad (3)$$

We compute the wake field for flat (KEKB-LER) and round (CERN SPS) beams. Micro-bunches are placed longitudinally every 0.1 mm and 1 mm for KEKB-LER and SPS, respectively. The macro-particles modeling the electron cloud are launched with a Gaussian distribution in the transverse plane. The initial velocities are set to zero.

The wake field is calculated for various sizes of the cloud maintaining a constant central density ( $\rho_c$ ) and increasing the total number of cloud electrons (line density  $\lambda_c$ ). The size of the electron cloud is characterized by two parameters ( $\Sigma_x, \Sigma_y$ ) referring to the horizontal and vertical size in units of the rms beam size ( $\sigma_x, \sigma_y$ ). For example, the line density  $\lambda_c$  for (a;b) is a**x** times that of (1,1).

We investigate the wake fields for KEKB-LER and CERN-SPS using the parameters of Table 1. Figure 1 depicts the horizontal and vertical wake forces obtained by the simulation for various electron cloud sizes and dipole field strength.  $Q$  is smaller for larger cloud size in the strong field case, which is same behavior as for the field-free case[6]. As expected, the dipole field strongly suppresses the horizontal wake and has a much weaker effect on the vertical wake. For the KEKB case, the strength of the horizontal wake for a 1T dipole field is about 2.3 times smaller than that for the field-free case. It is 4 times smaller in case of the SPS, which means the reduction of the horizontal wake by the dipole field is stronger in the SPS. Also at the SPS, the effect on the vertical wake is much smaller. The strong dipole field changes the horizontal wake frequency due to the combined effect of the magnetic and electric forces, where  $B_y$  is perpendicular to  $E_x$ . On the other hand, it does not change the vertical wake frequency, determined by the force between beam and photoelectrons alone. In a dipole magnetic field of  $B_y=1T$ , the Larmor frequency  $eB_0/m_e$  of an electron with energy 10eV is 18.2 GHz.

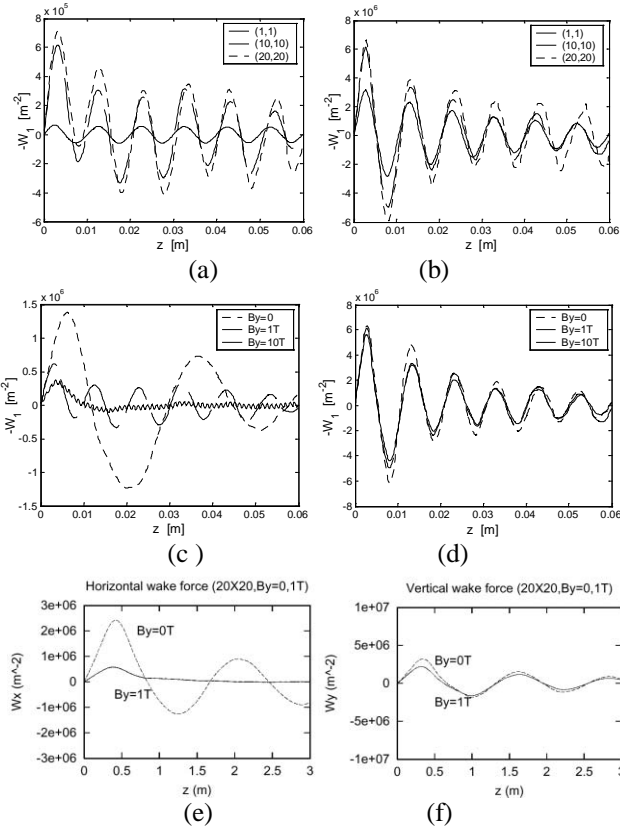


Figure 1: Wake field of electron cloud in dipole magnetic field. Horizontal (a) and vertical (b) wake for different cloud size dipole field strength 1T in the KEKB LER; Horizontal (c) and vertical (d) wake for different dipole field strength with cloud size=(10,10) in the KEKB LER; Horizontal (e) and vertical (f) wake for different dipole field strength with cloud size=(20,20) in the SPS;

Table 1: Basic parameters of the KEKB LER and CERN SPS

Variable	KEKB-LER	SPS
Particle type	$e^+$	$p$
Circumference	3016 m	6900m
Beam energy	3.5GeV	26 GeV
Bunch population	$3.3 \times 10^{10}$	$3.3 \times 10^{10}$
Rms beam size	0.42 mm/0.06mm	5mm/3mm
Bunch length	4mm	30cm
Average beta function	15 m	40 m

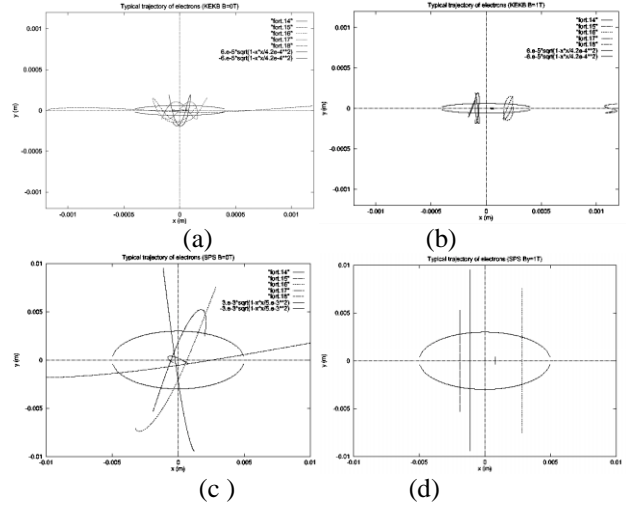


Figure 2 Typical drift orbit of photoelectron. (a)KEKB,  $B_y=0$ ; (b)KEKB,  $B_y=1T$ ; (c)SPS,  $B_y=0$ ; (d)SPS,  $B_y=1T$ .

### 3 FAST HEAD-TAIL INSTABILITY CAUSED BY THE WAKE FORCE IN DIPOLE FIELD

The transverse single bunch instability is analyzed using the perturbation theory of Vlasov equation as is done for impedance problems due to vacuum chambers and cavities. We first analyze the instability using the mode coupling theory [10]. We consider azimuthal mode coupling only for the lowest radial mode. The eigenvalue or tune of each mode is computed as a function of  $R_s/Q$ . The parameters are determined by fitting the simulated wake field to the expression

$$W(z) = c \frac{R_s}{Q} \frac{1}{\sqrt{1 - \frac{1}{4Q^2}}} \exp\left(-\frac{\omega_r}{2cQ} z\right) \sin\left(\frac{\omega}{c} z\right). \quad (4)$$

The strength of the wake field  $R_s/Q$  scales with the density of the electron cloud, since in our model each electron interacts with the beam independently. Table 2 gives the fitting parameters for the horizontal wake field of e-cloud in 1T dipole magnetic field according to Eq. (4). Figure 3 shows the computed mode-frequency variation as a function of  $cR_s/Q$  in both KEKB LER and SPS with constant  $\omega_r$  and  $Q$  taken from table 2. Since  $cR_s/Q$  is linearly related with the cloud density  $\rho_c = \lambda_c / (2\pi\sigma_x\sigma_y)$ , the figure also gives us the

dependence on  $\rho_c$ . The threshold of the horizontal mode-coupling instability is much higher than the operating value for both KEK LER and SPS. The vertical thresholds are about the same as for the field-free case [6].

Table 2 Parameters for the horizontal wake field induced by an electron cloud of density  $\rho_c=1.0 \times 10^{12}$  [m<sup>-3</sup>] in a 1T dipole magnetic field obtained by fitting the simulated wake to the resonator model.

	KEK LER	SPS
$\omega_r$ [s <sup>-1</sup> ]	$1.9 \times 10^{11}$	$1.0 \times 10^9$
$Q$	8.7	1.0
$cR_r/Q$ [m <sup>-2</sup> ]	$8.0 \times 10^5$	$5.0 \times 10^5$

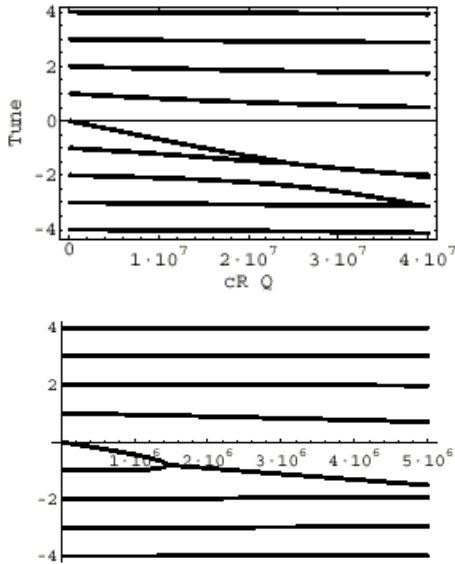


Figure 3: Horizontal mode coupling due to the e-cloud wake field in dipole magnetic field for KEK LER (above) and SPS (bottom)

Figure 4 shows the emittance growth due to the wake of electron cloud in a region with and without strong dipole field for the SPS obtained by direct simulation [15]. The offset of the proton bunch as a function time is also shown, in figure 5. As expected, the dipole field almost cancels any horizontal coherent motion and rms-size blow-up, and it also weakens the instability in the vertical direction. This is consistent with the above mode-coupling analysis using the simulated wake.

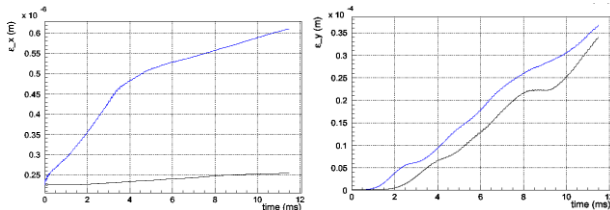


Figure 4 Emittance growth as a function of time due to the wake of electron cloud in a region with (black line) and without (blue line) strong dipole field in the SPS. Left: horizontal emittance; right: vertical emittance.

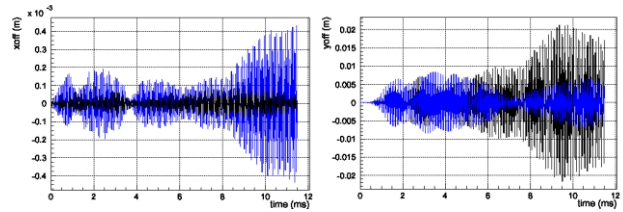


Figure 5 Beam centroid motion as a function of time due to the wake of the electron cloud in a region with (black line) and without (blue line) strong dipole field in the SPS. Left: horizontal; right: vertical.

### 3 SUMMARY

The wake due to the electron cloud in a dipole magnet field has been studied using a numerical code. The wake extracted from the simulation has been used to analyze the mode coupling instability in both KEK LER and CERN SPS. A direct simulation of the emittance growth has also been done for the SPS. Both approaches show that the dipole field can efficiently suppress the wake field or emittance growth in the horizontal direction. For a flat beam, there is little effect in the vertical direction as expected. For a round beam the vertical wake is also reduced by the dipole field due to the absence of the electron pinch in the horizontal direction.

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