ANALYSIS OF THE ION INDUCED COUPLED BUNCH INSTABILITY OBSERVED IN BEPC

Y. Luo Z.Y. Guo and C. Zhang

IHEP, P. O. Box 918-9, Beijing 100039, China

Abstract

With the single pass beam position monitor, an electron coupled bunch instability was observed in Beijing Electron-Positron Collider (BEPC), which could be explained as ion trapping. First the beam current threshold of the instability was calculated by the linear two-beam theory of ion trapping. Then a computer program based on weak-strong model was written to simulate the interaction between the ions and the beam, the tracking results successfully reproduced the betatron side bands observed experimentally, and gave a more reasonable growth time of the instability.

1. INTRODUCTION

In the fifth jointly photoelectron instability (PEI) by IHEP, China and KEK, Japan, a set of single pass beam position monitor (SPBPM) system was installed in the ring of the BEPC. This system was able to record bunch center's displacement bunch by bunch. For example, for BEPC, the RF harmonic number is 160, the adjacent bunches' interval is 2*ns*. In the series of experiments, 160 buckets were almost equally filled, the SPBPM system could record 16384 turns' data of bunch center's displacements. By doing FFT of these recorded data, the dipole oscillation information of the beam could be obtained. Also a synchrotron radiation light monitor was adopted to check the beam size change or beam coherent oscillations in the experiments.

In order to compare with the observation results between the position injection and electron injection modes, 160 equally populated electron bunches with total beam current 11.8*mA* were filled in the ring, under this condition the beam coherent oscillation was observed through the synchrotron radiation light monitor. The machine and beam parameters are given in Table 1.

Table 1: Machine and beam parameters

Circumference (m)	240.4
Energy (GeV)	1.3
Revolution Frequency f_0 (MHz)	1.247
Harmonic Number	160
Horizontal Working Point	5.82
Vertical Working Point	6.74
Average Hori. Beam Side(mm)	1.34
Average Vert. Bunch Size (mm)	0.277
Natural Emittance (mm mrad)	0.134

2. DATA PROCESSING

According to the coupled bunch instability theory [1], the frequencies of betatron side bands of the equidistant and equally populated multi-bunch beam are gave by

$$\omega_{p,\mu,Q_y} = (pM + \mu + Q_y)\omega_0 , \qquad (1)$$

Where M is the number of the bunches along the ring, p and μ are mode numbers, $-\infty < p < +\infty$, $\mu = 0,1,2....M-1$, Q_y is the vertical working point.

By doing FFT of the recorded bunch by bunch center's vertical displacements and picking out the relative amplitudes of all the betatron side bands, the side band distribution was obtained as shown in Fig. 1, where the horizontal coordinates is the multiples of the revolution frequency, the absolute values of vertical coordinates are the relative amplitudes of the side bands close to the multiples of revolution frequency, the upper side bands were drawn in the lower half plane, the lower side bands in the upper half plane.

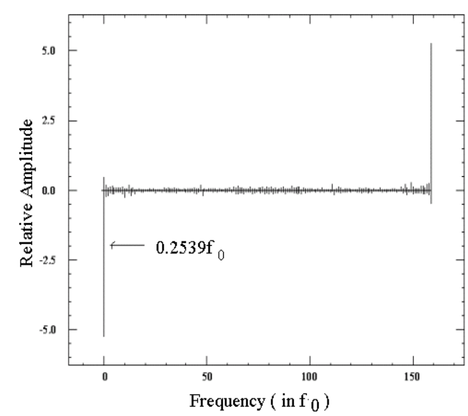

Figure 1: Side band distribution by doing FFT of the recorded data with SPBPM

From Fig. 1, it can be seen that there is only one upper side band whose amplitude is much higher than other's, which centered at 0.2539 f_0. So the coupled bunch oscillation is mainly represented by this frequency or this mode, whose mode number is 153 from eq.(1). What is the most important is that the side band represented an unstable coupled motion since the nominal vertical working point is 6.74. In the following explanation one can see that the side band was due to ion trapping.

3. LINEAR TWO BEAM THEORY EXPANALATION

Under the assumption of the linear two-beam theory of ion trapping [2], the ion's oscillation angular frequency is given by

$$\omega_{i,y} = \left[\frac{2\lambda_e r_p c^2}{A\sigma_{e,y}(\sigma_{e,x}+\sigma_{e,y})} \right]^{1/2}, \qquad (2)$$

The dispertion relation as

$$(\Omega^2 - \omega_{i,y}^2)[(\Omega - n\omega_0)^2 - \omega_{e,y}^2 - Q_y^2\omega_0^2] = \omega_{i,y}^2\omega_{e,y}^2, \quad (3)$$

where

$$\omega_{e,y}^2 = \frac{2\lambda_i r_e c^2}{\gamma\sigma_y(\sigma_x+\sigma_y)} = \frac{Am_p}{\gamma m_e}\eta\omega_{i,y}^2, \qquad (4)$$

Ω is coupled oscillation frequency between the ion and the beam, Q_y the beam's vertical working point, γ the beam relativity energy factor, η the ion neutralization factor, m_e and m_p the masses of electron and proton respectively.

Defining $x = \frac{\Omega}{\omega_0}, \nu_e = \frac{\omega_{e,y}}{\omega_0}, \nu_i = \frac{\omega_{i,y}}{\omega_0}$, eq.(2) can be rewritten as

$$(x^2 - \nu_i^2)[(x-n)^2 - \nu_e^2 - Q_y^2] = \nu_i^2\nu_e^2 \ . \qquad (5)$$

So under a certain beam current, for a mode number *n* to solve the above equation, if one root's imaginary part is positive, the beam-ion system is unstable. Since the coefficients of eq.(5) are all real, the imaginary roots should be paired, so if the imaginary part of one root is not equal to zero, the beam is not stable, the growth rate τ_g^{-1} is given

$$\tau_g^{-1} = \omega_0 |\mathrm{Im}(x)| . \qquad (6)$$

For the above specific experimental condition, assuming neutralization factor 0.001, mode number *n*=7, solving eq.(5) for different beam current, the dependence of the real part of Ω/ω_0 and $\omega_{i,y}/\omega_0$ vs. beam current are shown in Fig. 2, the imaginary part of Ω/ω_0 vs. beam current in Fig. 3.

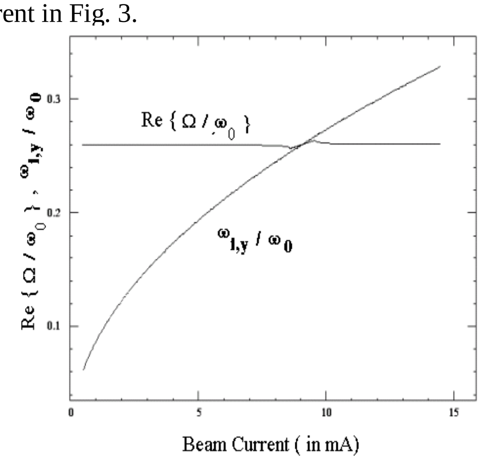

Figure 2: Re{Ω/ω_0}, $\omega_{i,y}/\omega_0$ vs. beam current

From Fig. 2, when the beam current is between 8.6~ 9.6*mA*, which is close to experimental current 11.8*mA*, the beam oscillation frequency is almost equal to that of the ion beam, both about 0.26 f_0. So at this current there should be one peak centered at this frequency in the beam spectrum, which is very close to the observed highest side band's frequency.

And from Fig. 3 at this beam current the imaginary part of Ω/ω_0 is not equal to zero, so the system is unstable in dipole mode oscillation. This can explained why from the synchrotron light monitor the light spot was unstable. From eq.(6), the growth time was given about $0.035ms$, which is much faster and so unbelievable.

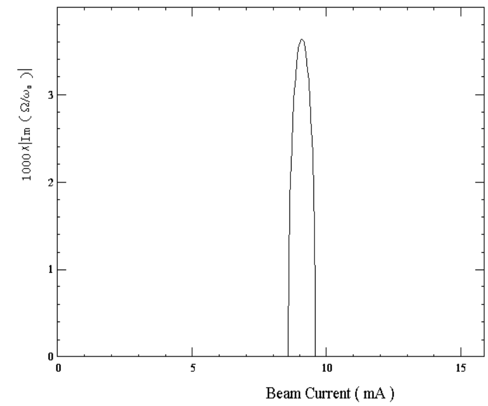

Figure 3: |Im{Ω/ω_0}| vs. beam current

4. PROGRAM SIMULATION

Since the linear two-beam theory of ion trapping assumes the linear interaction between the ion beam and electron beam, and it is also not sufficient to look into the side bands etc. Now based on the weak-strong model [3], a simulation program was written and used for the tracking of interaction between the ions and bunches.

The ions in the program are represented by macro-particles. They are assumed to be located only at several locations along the ring, usually those locations are equally distributed along the ring. The bunches are assumed to be rigid three dimensional gaussian distributions, which couldn't be influenced by the forces exerted by the ions. At one ion location, when a bunch passes by, all the ions interact with the bunch, while no bunch passing by, the ions shift until hitting the pipe wall and lost. The center of the rigid bunch transfers according to the linear magnet transfer matrix.

One ion's velocity change $\Delta v_{x,y}$ when a bunch passing by is

$$\Delta v_y + i\Delta v_x = -\frac{n_e r_p c}{A} \cdot \sqrt{\frac{2\pi}{(\sigma_{e,x}^2 - \sigma_{e,y}^2)}} \cdot f(x,y), \quad (7)$$

where n_e the electron particle number in the bunch, *A* the mass number of the ion, *x* and *y* the ion's displacements with respect to the center of the bunch, *f(x,y)* is dimensionless function related to the bunch sizes,

$$f(x,y)=w\left(\frac{x+iy}{\sqrt{2(\sigma_{e,x}^2-\sigma_{e,y}^2)}}\right)-\exp\left(-\frac{x^2}{2\sigma_{e,x}^2}-\frac{y^2}{2\sigma_{e,y}^2}\right)$$

$$w\left(\frac{x\frac{\sigma_{e,y}}{\sigma_{e,x}}+iy\frac{\sigma_{e,x}}{\sigma_{e,y}}}{\sqrt{2(\sigma_{e,x}^2-\sigma_{e,y}^2)}}\right) \tag{8}$$

w(x+iy) is the comples error function. On the other hand the angle change of the bunch due to one ion is given

$$\Delta y'+i\Delta x'=\frac{r_e}{\gamma}\cdot\sqrt{\frac{2\pi}{\left(\sigma_{e,x}^2-\sigma_{e,y}^2\right)}}\cdot f(x,y) \tag{9}$$

where r_e is the classic radius of electron.

In present simulation, four equidistant ion locations are assumed. This program also has considered the process of CO^+ production. All the ions created in one revolution time are equally distributed among the four ion production locations.

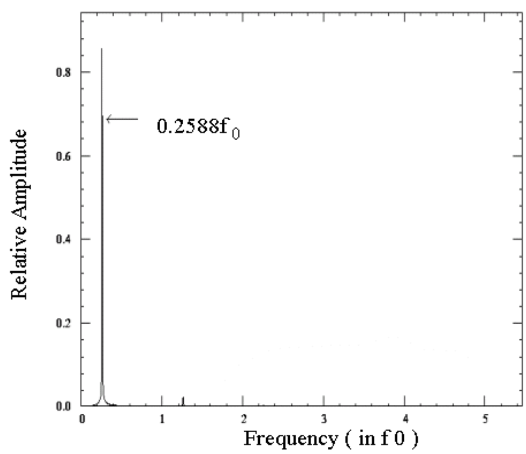

Figure 4: The beam spectrum in the low frequency by tracking

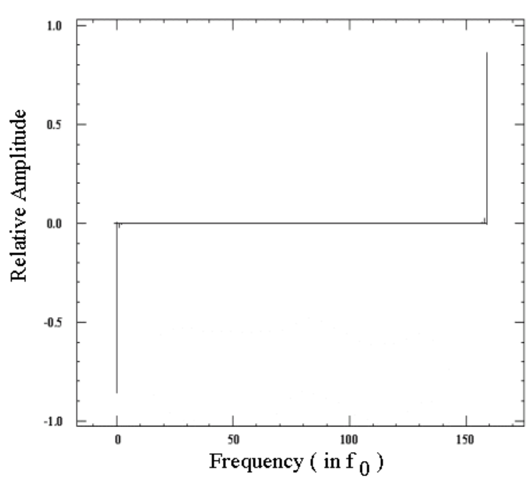

Figure 5: The full span side bands distribution by tracking

In order to shorten the computation time for the experimental condition, the tracking pressure of CO is assumed to be *1.0e-07Torr*, much higher than the practical pressure. With 160 equally populated bunches with total beam current $11.8mA$, track 2048 turns and record every bunch center's vertical displacements. Doing FFT of these data, Fig. 4 shows the low frequency part of the beam spectrum, from which only one upper side band's amplitude is much higher than other's, it is allocated at $0.2588 f_0$ coinciding with the experimental result. Fig. 5 gives the full span distribution of all the side bands. It can be seen that the simulation results are much close to the experimental results Fig. 1, the simulation program successfully reproduces the side bands.

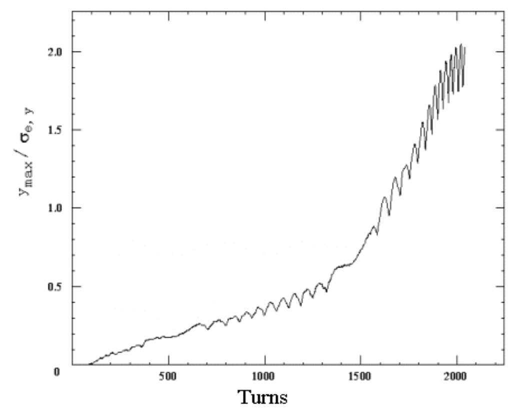

Figure 6: The maximum vertical displacements $y_{\max}/\sigma_{e,y}$ vs. turn

Fig.6 shows the maximum vertical displacement of bunches vs. tracking turn, by doing exponential fitting, the growth time is given as $0.50ms$. In order to get the growth time's dependence with the CO pressure, a set of tracking has been done for 160 equally populated bunches and total current $80.0mA$ with different CO pressure, from which a practical formula has been achieved,

$$\tau_g\cdot p_{co}^{0.584}=C \tag{10}$$

where C is a current dependent constant, τ_g is the growth time in revolution period, p_{CO} is the CO pressure in $nTorr$. In the practical experiment, the average vacuum pressure is about $0.5nTorr$, for BEPC vacuum condition, the CO takes up 15% of the residual gas, so the practical CO pressure is about $0.075nTorr$. With the growth time under CO pressure $1.0\times10^{-7}Torr$, assuming eq.(10), one gets the growth time due to the ion trapping about $33.4ms$ under the experimental condition, which is much more reasonable since the radiation damping time is about $86ms$.

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[2] Bocchetta C. Lifetime and Beam Quality. In: Turner S ed. CERN 98-04. 1999. p272-278

[3] Ohmi K. Phys. Rev. E, 1997, **55:**7750