PROFILE OF LOSS CONE AND MAGNETIC STRESS ON RESONANCE SURFACE OF THE 14.4 GHz ECRIS

M. H. Rashid* and R. K. Bhandari
Variable Energy Cyclotron Centre (DAE), Sector-1, Block-AF, Bidhan Nagar, Kolkata-700 064, INDIA

Abstract

Recently a 14.4 GHz ECRIS [1] was designed for the superconducting cyclotron under construction. It has min-B (Ioffe) configuration for reflecting the escaping electrons. The electrons gyrating around a line of force falling in a loss cone (LC) will not be reflected by the mirror field. The equation for evaluating the LC is formulated and used for LC calculation from the MR at a point. The MR thereby giving the LC profile around the resonance surface is evaluated. The components of the magnetic stress dyad also were calculated.

1 LOSS CONE

An LC signifies the loss of electrons falling into it unless it is knocked out of it by some physical processes. So, it is important to evaluate it quantitatively which gives qualitative idea of the loss of electrons all around the plasma chamber. It gives qualitative idea because its motion is vigorously affected by many other physical phenomena like collision, diffusion etc. So for the first time an attempt has been made to calculate it on a surface just enclosing the resonance zone and evaluate the reflection co-efficient of electrons at a point of its creation on the surface and subsequent motion.

1.1 Formulation

We assume a line of force QSOF in the plasma chamber which encounters the cylindrical wall or end of the chamber in both the forward and backward direction of the line (Fig. 1a). Let an electron gyrates around the line of force at O with total momentum $P = |\mathbf{P}|$ having components parallel and perpendicular to the line of force $P_{\|}$ and $P_{\perp}$ respectively. A remarkable point is that the electrons may have $P_{\|}$, parallel or antiparallel i.e. it may move forward (in positive direction) or backward (in negative direction) w. r. t. the line of force. The magnetic field at the two points where the lines of force meet the wall may be different and represented by B_{s+} and B_{s-} in parallel and antiparallel direction respectively. The local magnetic field at O is B_o and B_{min} is the minimum field at certain point on the line of force. The equations (1a) and (1b) give the local MRs in forward and backward directions respectively at the point O whereas equation (1c) give the R in common use so far defined traditionally. The function max (B_{s+} , B_{s-}) in equation (1c) selects maximum of the two values.

$$R_+ = B_{s+} / B_o \qquad \text{(1a)}$$

$$R_- = B_{s-} / B_o \qquad \text{(1b)}$$

$$R_m = max(B_{s+}, B_{s-}) / B_{min} \qquad \text{(1c)}$$

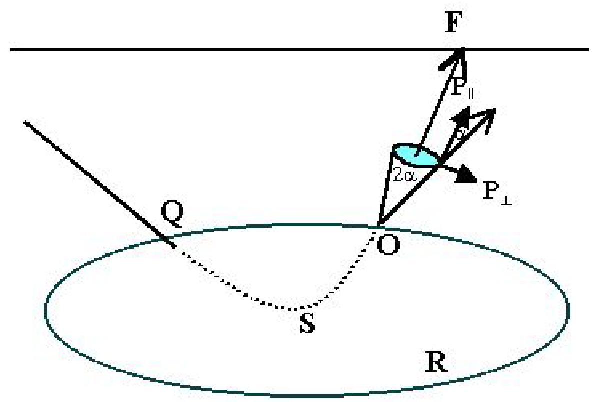

Figure 1(a): An electron is created at O on the line of force and moves around OF at an angle α.

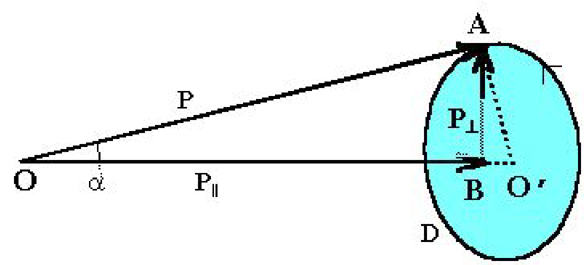

Figure 1(b): The shaded elliptical portion is a part of the assumed spherical surface with radius P (magnitude of momentum). The line of force passes along the line OO' (through the centre of the shaded dish).

The MR, R_m gives a qualitative feeling of the tightness of the plasma confinement and the efficiency of the multiply stripped ion production in the plasma. But as far as the production of electron at certain position and subsequent motion of the electron is concerned the concept of local MRs, R_+ and R_- are very important for evaluation and estimation of electron reflection or loss.

* E-mail: haroon@veccal.ernet.in

The actual angle of the cone of spiralling electron around the line of force and apex angle of the LC is given by equation 2a and 2b respectively.

The solid angle of the LC is deduced from figure (1b). Let an electron is at O and taking the magnitude of the momentum of electron as radius we imagine a sphere around O. The circular shaded portion enclosed by the gyrating electron is a part of the sphere. The area (S) of the shaded dish and the solid angle of the electron is given by equations 3 and 4a respectively. Using the apex angle obtained by the adiabatic invariance of the magnetic moment of electron we obtain the solid angle of the LC given by equation 4b.

$$\alpha = \sin^{-1}\left(P_\perp / P\right) \quad \text{---------------------------} (2a)$$

$$\alpha_{apex,i} = \sin^{-1}\left(1/\sqrt{R_i}\right) \quad \text{----------------------} (2b)$$

$$S = 2\pi P(P - P_\parallel) = 2\pi P^2(1 - \cos\alpha) \quad \text{------} \quad (3)$$

$$d\Omega = 2\pi\left(1 - \cos\alpha\right) \quad \text{-------------------------} (4a)$$

$$d\Omega_i = 2\pi\left(1 - \cos(\alpha_{apex,i})\right) \quad \text{----------------} (4b)$$

where, $R_i \in \{ R_+, R_-, R_m \}$. That is, the subscript i = +, – or m.

If $\alpha < \alpha_{apex,i}$ then the electron falls inside the LC and there is sound probability of being lost unless it switches over to another position and starts moving from the point out of the LC afresh.

1.2 Evaluation

The MRs, R_+, R_- or R_m at any point in the chamber are given by TrapCAD [2,3] on the line of force passing through the point after properly feeding the coil and multipole magnetic field data. If the local field at the point, we assume the field (B_o) at point O, is known then all the fields B_{s+}, B_{s-} and B_{min} can be calculated readily using the obtained values from above formulae of MRs.

The apex angle $\alpha_{apex,i}$ and solid angle $d\Omega_i$ of the LC were evaluated at the vicinity of the resonance surface (Fig. 2) in the chamber corresponding to the *RF* frequency 14.4 GHz. The resonance surface is positioned from –6.0 cm to 6.0 cm w. r. t. the centre. The values of $d\Omega_i$ were evaluated in the parallel and anti-parallel directions of the lines of force along the whole plasma length and from 30° to 90° azimuth due to field symmetry (Fig. 3 and 4). The sum of the two gives the total solid angle ($d\Omega_t = d\Omega_+ + d\Omega_-$) at a point. The LC was evaluated using the traditional MR, R_m also (Fig. 5) inside the plasma chamber.

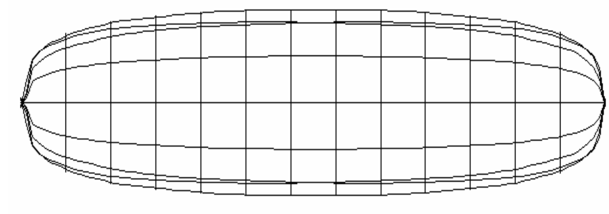

Figure 2: The resonance field surface at 5.143 kG have 3.5 cm diameter at the mid-length. The centre meridian is at 0^0 along the length and others at 30^0 gap.

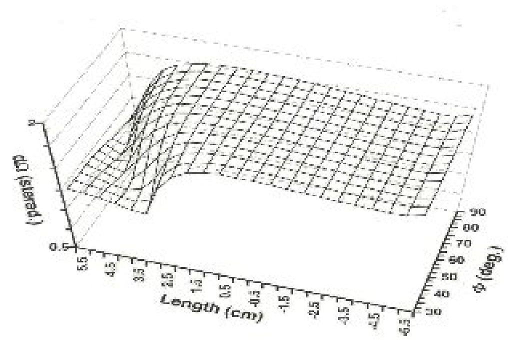

Figure 3: The LC plot for parallel motion of electrons w.r.t. the lines of force (i.e. using MR, R_+).

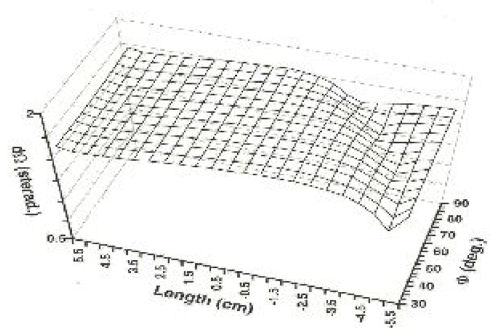

Figure 4 : The LC plot for anti-parallel motion of electrons w.r.t. the lines of force (i.e. using MR, R_-).

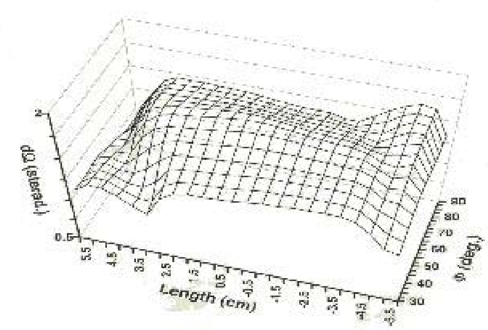

Figure 5: The LC plot for motion of electrons using traditional MR, R_m.

1.3 Loss of plasma in LC

We assume a plasma having two components one inside the LC (gas A) and the other outside it (gas B) [4] for estimating the loss of particle falling into the LC due to particle-particle collisions. The probability of loss of gas-B is given by equation 5.

$$p_1 = \tfrac{1}{2}\left(1-\sqrt{1-1/R}\right) \quad (5)$$

The probability of transfer of particle gas-A → gas-B due to large angle collision is given by equation 6.

$$p_2 \cong (1+\ln R)^{-1} \quad (6)$$

So the resultant transfer and loss rate is given by (7).

$$N = n\left(p_1/\tau_1 - p_2/\tau_2\right) \text{ ions/m}^3\text{sec} \quad (7)$$

Where n and τ_1, τ_2 are gas density and mean collision times respectively.

The field configuration is 3-fold symmetric along the azimuth because of extupole radial field. The loss holes are formed at 90^o, 210^o and 330^o azimuth at the extraction side for electrons moving towards the negative direction of the field i. e. towards the extraction. The azimuthal position of the loss hole is rotated by 60^o at the injection side for the electrons moving towards the positive direction of the field i. e. towards the injection due to the presence of reverse radial field component of the axial field at the extraction side. But these loss hole positions are far away from the resonance field surface at the ends and one should not be alarmed with it.

It is seen from the figures 3 and 5 that the LC at the mid-length of the plasma surface are more or less constant while using the local MRs and the electrons have least LC while moving towards the mirror peak after scattering or creation. It is apparent from the calculation that the electrons which start moving from inside the resonance surface has smaller LC than the electrons starting motion from outside the surface. One can easily find the position of the magnetic hole where maximum loss of the electrons take place (Fig. 5).

2 MAGNETIC STRESS

The plasma generated by the energetic electrons is contained in the external magnetic field and it is adequately described by means of the single magnetic fluid-static model. The plasma current **J** and external magnetic field **B** both lie on surfaces of constant pressure [5]. If the system is to be confining, then the surface in which B lies must be closed and nested. If the Lorentz force **J**x**B** is directed to the z-axis then the plasma pressure also increases in this direction. A plasma may be kept from the chamber wall if on the limiting plasma pressure *P*=0. The line among the enclosed magnetic surfaces where plasma attains maximum pressure is called the magnetic axis. We have:

$$-\overline{\nabla}P + \left(\overline{\nabla}\times\overline{B}\right)\times\overline{B} = -\overline{\nabla}P + \overline{\nabla}\cdot \mathcal{T} = 0 \quad (8)$$

$$\overline{\nabla}\cdot \mathcal{T} = -\overline{\nabla}\left(\frac{B^2}{2\mu_0}\right) + \left(\overline{B}\cdot\overline{\nabla}\right)\overline{\nabla} \quad (9)$$

Where the bar or the bold face represent the vectors.

From Eq.8 and 9 one gets the Virial equation on expanding it. For the simplest equilibrium configuration when the magnetic lines are the set of the parallel lines, the 2^{nd} term on RHS of Eq. 9 vanishes. In that case equilibrium condition of the plasma can be expressed as:

$$P + \frac{B^2}{2\mu_0} = P_0 + \frac{B_0^{\,2}}{2\mu_0} = const. \quad (10)$$

where P and P_0 are the plasma pressure inside and outside a plasma column depending on the specific plasma density and temperature there.

In conventional ECR longitudinal tandem mirror and radial multipole magnetic field are present, so in addition to the pressure described by Eq. 10 the magnetic shear forces are also present. The magnetic stress dyad $\mathcal{T}$ consists of magnetic stress (tension) as well as the shear (off-diagonal) terms and can be written as:

$$\mathcal{T}_{ij} = \frac{1}{\mu_0}\left(B_iB_j - \delta_{ij}\frac{B^2}{2}\right) \quad (11)$$

where, i,j∈ { x , y , z } and $\delta_{i,j}$ is the kronecker delta.

Once the field components are known the $\mathcal{T}_{ij}$ can be calculated. The magnetic field on the resonance surface is 5.143 corresponding to the 14.4 GHz frequency and magnetic pressure $1.05\text{x}10^5$ N/m^2. The magnetic field components [1] at grid-points formed by the latitudes and longitudes on the resonance surface (Fig.2) in cylindrical polar co-ordinate were evaluated and after changing the components into the rectangular co-ordinate the magnetic pressure dyad components were evaluated.

REFERENCES

[1] M. H. Rashid et. al., Intl. Conf. on Cyclotron & their..., East Lansing (MI), USA, CYCLOTRON2001.
[2] J. Vamosi and S. Biri, Nucl. Instr. and Meth., **B94**, (1994), 297.
[3] S. Biri and J. Vamosi, Proc. of the EPAC 94, Vol.-2, (1994), 1432.
[4] J. G. Linhart, North-Holland Publishing Company, Amsterdam, (1961), 214.
[5] A. Sitenko, Plasma Physics Theory, Chapman & Hall, London, (1995), 71.