A NEW METHOD FOR DISPERSION CURVES OF HOM IN PERIODICAL AXISYMMETRIC ACCELERATING STRUCTURES

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Abstract

A new method for dispersion curves of high order mode (HOM) in periodical axisymmetric accelerating structures is developed. It discards the view of circuits completely and assumes that fields of HOM can be expanded with normalized orthogonal open and short modes in each cavity. With properties of these modes on the boundaries considered, the dispersion curves are available. An example is shown in this paper.

1 INTRODUCTION

Both the circuit methods and the field methods can usually be used to analyse the dispersion curves in the axisymmetric accelerating structures. In this paper a new field solver is presented, with which we can obtain the curves of the high order modes (HOM) in these structures effectively.

It is assumed that the fields in the whole periodic structure can be written as the linear sum of the so-called open and short modes, which are constructed on the basis of the resonant modes in a single cavity with different boundary conditions. Then it is possible for us to obtain the dispersion curves with some mathematic manipulations. The potential usage of the method is to analyse HOM in the non-periodic accelerating structures in the following work.

2 EXPRESSIONS OF THE DISPERSION CURVES

2.1 Open and short modes

The definitions of the open and short modes are explained well by R.M.Bevensee[1]. For the ordinary disk-loaded waveguide shown in Fig 1, the short modes are the normalized orthogonalized resonant ones when the beam holes are electric-shorted in a single cavity, while the open modes are correspondent to the magnetic-shorted holes. Let $\{\overline{E}_i\}$ and $\{\overline{e}_i\}$ describe the open- and short-mode sets respectively. Their eigenfrequencies are expressed by Ω_i and ω_i. Here only the electric fields are taken into accounts.

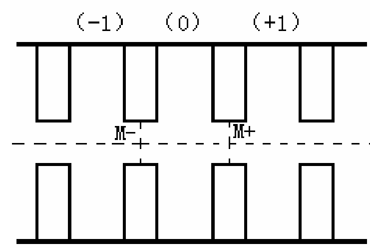

Fig 1 Disk-loaded waveguide

2.2 Expansion of the Fields

With all passbands considered, the electric field of an arbitrary phase (φ) in the disk-loaded waveguide is supposed to be expanded as:

$$\mathbf{E}(\omega) = \sum_i D_i(\omega)\overline{E}_i + \sum_i d_i(\omega)\overline{e}_i \quad . \qquad (1)$$

where ω is the frequency corresponding to φ.

It must be indicated that the basis is not orthogonal in general, though $\{\overline{E}_i\}$ and $\{\overline{e}_i\}$ are orthogonalized respectively. However, the most important is that the basis is complete and no potential functions are needed to describe the travelling wave fields.

Since the electric fields are complex usually, the expansion coefficients should be complex in most cases. It is required that the field satisfies the wave equation $\nabla\times\nabla\times\mathbf{E} = k^2\mathbf{E}$ and the Floquet boundary condition on the beam holes according to the Maxwell theory. We treat here only those structures in which the phases of the fields in all passbands of interests tend to shift abruptly across the coupling holes ($M\pm$). Then the tangential and normal electric field on $M\pm$ can be written as:

$$\mathbf{E}_t\,|_{M\pm} = d_i\overline{e}_i^{\,0} = \frac{d_i}{2}(\overline{e}_i^{\,0} + e^{\mp j\varphi}\overline{e}_i^{\,+1})\,|_{M+} \quad , \qquad (2)$$

$$\mathbf{E}_n\,|_{M\pm} = D_i\overline{E}_i^{\,0} = \frac{D_i}{2}(\overline{E}_i^{\,0} + e^{\mp j\varphi}\overline{E}_i^{\,+1})\,|_{M+} \, . \qquad (3)$$

Since the open and short modes in a longitudinal symmetric cavity must be also symmetric, we say that fields of even symmetry have E_z components that suffer an even number of reversals while fields of odd symmetry have E_z components that suffer an odd number of reversals across a cavity.

2.3 Expressions of the Dispersion Curves

Substitute the field expansion above and the expressions on the holes into the wave equation. Use the standard Galerkin technique and the second Green theorem[2]. Considering the parities of the open and short modes, we can get the following equation, which shows the relationship between k $(k=\omega\sqrt{\mu\varepsilon})$ and the phase shift φ:

$$AC = k^2 BC \quad , \tag{4}$$

where C is combined with the field coefficients that need to be determined. A and B are the square matrixes whose elements can be computed by:

$$A=\begin{bmatrix} A_{ss}^{ee} & 0 & A_{so}^{ee} & A_{so}^{eo} \\ 0 & A_{ss}^{oo} & A_{so}^{oe} & A_{so}^{oo} \\ A_{os}^{ee} & A_{os}^{eo} & A_{oo}^{ee} & 0 \\ A_{os}^{oe} & A_{os}^{oo} & 0 & A_{oo}^{oo} \end{bmatrix} \quad , \tag{5}$$

$$B=\begin{bmatrix} I & 0 & B_{so}^{ee} & B_{so}^{eo} \\ 0 & I & B_{so}^{oe} & B_{so}^{oo} \\ B_{os}^{ee} & B_{os}^{eo} & I & 0 \\ B_{os}^{oe} & B_{os}^{oo} & 0 & I \end{bmatrix} \quad . \tag{6}$$

I is defined as the unit matrix. The superscripts e represents the even modes while o represents the odd modes. The subscripts s and o indicate the short and open modes respectively. The sub blocks in the matrix equations are available with the following relations:

$$(A_{ss}^{ee}, A_{ss}^{oo}) : a_{ij} = P_i P_j \delta_{ij} \quad , \tag{7}$$

$$A_{so}^{ee} : a_{ij} = P_i^2 T_{ij} - (1-\cos\varphi) M_{ji} \quad , \tag{8}$$

$$A_{so}^{eo} : a_{ij} = P_i^2 T_{ij} - i\sin\varphi M_{ji} \quad , \tag{9}$$

$$A_{so}^{oe} : a_{ij} = P_i^2 T_{ij} + i\sin\varphi M_{ji} \quad , \tag{10}$$

$$A_{so}^{oo} : a_{ij} = P_i^2 T_{ij} - (1-\cos\varphi) M_{ji} \quad , \tag{11}$$

$$A_{os}^{ee} : a_{ij} = p_i^2 T_{ji} + (1+\cos\varphi) M_{ij} \quad , \tag{12}$$

$$A_{os}^{eo} : a_{ij} = P_i^2 T_{ji} - i\sin\varphi M_{ij} \quad , \tag{13}$$

$$A_{os}^{oe} : a_{ij} = P_i^2 T_{ji} + i\sin\varphi M_{ij} \quad , \tag{14}$$

$$A_{os}^{oo} : a_{ij} = P_i^2 T_{ji} + (1-\cos\varphi) M_{ij} \quad , \tag{15}$$

$$(B_{so}^{ee}, B_{so}^{eo}, B_{so}^{oe}, B_{so}^{oo}) : a_{ij} = T_{ij} \quad , \tag{16}$$

$$(B_{os}^{ee}, B_{os}^{eo}, B_{os}^{oe}, B_{os}^{oo}) : a_{ij} = T_{ji} \quad . \tag{17}$$

It is not difficult to obtain the parameters (P_i, p_i, T_{ij}, M_{ij}) from the frequencies and fields of the open and short modes:

$$P_i = \Omega_i \sqrt{\mu\varepsilon}\ ,\ \ p_i = \omega_i \sqrt{\mu\varepsilon} \quad , \tag{18}$$

$$T_{ij} = \int_V \bar{E}_i \bullet \bar{e}_j dV \quad , \tag{19}$$

$$M_{ij} = \int_{M-} \bar{e}_i \times \nabla \times \bar{E}_j \bullet \hat{z} dS \quad . \tag{20}$$

The problem has been turned into a generalized eigen-value equation. It is easy to be solved with today's mathematic technique and computers. Then the dispersion curves of interests and the corresponding fields are available to describe the whole periodic accelerating structures.

3 AN EXAMPLE

An example for the first two curves of the dipole modes in the disk-loaded waveguide with round corners (Fig 2) is given here. The open and short modes are computed with PISCES-II [3] and another finite element code CAFE [4] precisely. The results are shown in Fig 3. The accuracy is believably acceptable from the previous results of the basic passband [5].

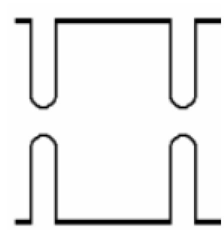

Fig 2 The waveguide with round corners disk-loaded
（t=2.0mm, D=8.7474mm, b=10.77871mm, a=4.5mm）

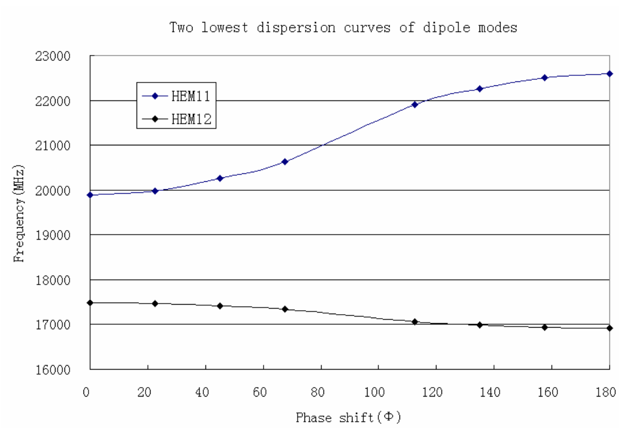

Fig 3 Two lowest dispersion curves of dipole modes
in the disk-loaded waveguide

4 CONCLUSIONS

Besides the dispersion curves, the fields in the whole structure can also be obtained when the expansion coefficients are computed as the eigenvectors of the matrix equation.

It has been verified that this method can solve HOM in the periodic accelerating structures effectively. It is also expected to be used to analyse the non-periodical structures in the further study.

5 ACKNOWLEDGEMENTS

We owe our great thanks to Dr. V.V.Paramov from INR for his patient and beneficial discussions. His suggestions helped us to smooth away a lot of difficulties. We also thank Prof. Takata a lot. He provided one of the authors with the opportunity to visit KEK and finish our work there. Thank Mr. Iwashita for his authorization of PISCES-II.

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